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Nuclear Instruments and Methods in Physics Research A 415 (1998) 455–459

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

Simulation studies of transverse resonance effects in space-charge-dominated beams[1]

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Abstract

Particle-in-cell simulations exploring the effects of transverse envelope resonances on the emittance of space-charge-dominated beams have been carried out using a new 2-D transverse "slice" model in the WARP particle-in-cell code. In this paper, we consider resonances between an applied quadrupole field error and harmonics of the asymmetric mismatch mode of beam envelope oscillation. We examine both acceleration and bunching, and show that rapid passage through a major resonance significantly reduces the attendant emittance growth. Published by Elsevier Science B.V.

PACS: 29.20. − c; 29.27. − a; 52.65.Cc; 52.65.Rr

Keywords: Resonance; Mismatch; Recirculator; Storage ring; Simulation; Particle-in-cell

1. Introduction

Two general classes of transverse resonance effects are significant in circular particle accelerators and storage rings: coherent effects, which are well-described by moment (envelope) equations, and incoherent effects, which require a kinetic model such as a particle-in-cell (PIC) simulation. The beam centroid oscillates coherently at approximately the undepressed betatron frequency (phase advance per lattice period) σ_0 minus an image-force correction. In the smooth approximation the frequency of the symmetric mismatch (envelope) mode is $\omega_{\mathrm{sym}} \approx \sqrt{2\sigma_0^2 + 2\sigma^2}$ and that of the asymmetric mismatch mode is $\omega_{\mathrm{asym}} \approx \sqrt{\sigma_0^2 + 3\sigma^2}$ (here σ is the depressed phase advance) [1]. Such envelope oscillations can resonate with errors in applied fields, including monopole errors (symmetric perturbations of the beam ellipse semi-axes a, b), dipole

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[1] Work performed under the auspices of the U.S. Department of Energy by LLNL and NRL under contracts W-7405-ENG-48, DE-AI05-92ER54177, and DE-AI05-83ER40112.

0168-9002/98/$19.00 Published by Elsevier Science B.V.
PII: S0168-9002(98)00419-7

errors (or quadrupole offsets) which perturb the beam centroid, and quadrupole strength errors (asymmetric perturbations of a, b). Incoherent effects include emittance growth driven by mismatch oscillations, and halo generation. Coherent and incoherent effects are in fact coupled, and so the division is somewhat arbitrary.

In the induction recirculator approach to a Heavy-Ion Fusion driver, the intent is to minimize resonance effects by rapidly accelerating the beam "through resonances" – the beam is "different on every lap" and errors are expected to add as in a long linac – using beam steering to correct the centroid motion. In the storage-ring approach, rapid bunching of the beam causes shifts in the resonances due to the increasing space charge density and image effects. In all these cases there is a spread of individual-particle "tunes," and the particles' motion differs greatly from their motion in the absence of space charge.

A variety of studies have been carried out using the PIC "slice" model WARPxy. In this model, particles are described by their transverse coordinates x, y, and all three momenta p_x, p_y, p_z, all of which are advanced step-by-step along the machine axis z. One set of studies examined the effects of misaligned quadrupole magnets and dipole strength errors, looking at resonances between a harmonic of the betatron oscillation frequency and an error in the applied field. In those studies the predominant motion observed was that of the centroid; little emittance growth was evident until the displacement had grown very large, in agreement with the work of Hofmann [2].

Another class of WARPxy-based studies considered quadrupole strength errors, and is discussed in this paper. Coasting beams show emittance growth and halo formation varying with their proximity to resonance. Accelerating beams undergo enhanced emittance growth as they pass through the resonance; rapid acceleration through the resonance reduces the growth. Rapid pulse compression behaves similarly.

Transverse resonance effects in the presence of strong space charge have been studied by others experimentally [3], theoretically [4,5], and via simulations [2,6]; see also references cited therein.

2. The simulations

We began by carrying out a set of envelope calculations to learn what classes of behavior might be expected. Using parameters relevant to the nominal operating regime of the LLNL small recirculator, we followed both initially matched and initially mismatched K^+ (mass 39 amu) beams of energy 80 keV over 300 periods of a FODO lattice of sharp-edged magnetic quadrupoles, with the half-lattice period $L = 36$ cm and the quadrupole occupancy $= 0.305$. With $\sigma_0 = 7.45^\circ$ and $\sigma = 17.2^\circ$, 1.66 mA, $a_0 \sim 1.4$ cm, $b_0 \sim 0.76$ cm, the smooth approximation referred to earlier gives $\omega_{\text{sym}} \approx 112^\circ$ and $\omega_{\text{asym}} \approx 83^\circ$. A power spectrum of one semi-axis of the actual envelope solution for the FODO system displays peaks centered at $\omega_{\text{sym}} \approx 117^\circ$ and $\omega_{\text{asym}} \approx 90^\circ$, with some spread arising from the system's nonlinearity. We then applied a 1% quadrupole strength error once every "lap of the ring", i.e. once every 20 lattice periods (l.p.'s); such an error is unrealistically large, but was chosen to make the effects being studied more clearly visible. The error induced unstable growth of the asymmetric mismatch mode, so that after 300 l.p.'s the peak values of a and b of an initially matched beam oscillated from 0.94 to 1.8 cm. In this case there is a resonant drive because the perturbation period is five times that of the mismatch mode. Note that the "seed" of the mismatch is a result of the beam's first encounter with the applied error. In contrast, when we applied the same error twice as often, we observed little or no growth in the mismatch oscillations even with an initial mismatch; this can be understood because the applied perturbation period is 2.5 times that of the mismatch mode, and so the mismatch is driven with alternating sign on each encounter.

Mismatch oscillations remain undamped in an envelope model. When the full kinetic PIC model is employed (with a semi-Gaussian initialization), the quantities $2x_{\text{rms}}$ and $2y_{\text{rms}}$ initially track the envelope model's a and b, but then their oscillation (mismatch) amplitude typically diminishes due to phase mixing. There is a concomitant increase in emittance growth as the energy in the mismatch oscillations is "thermalized". This effect is shown in Fig. 1 for a beam perturbed every 4 l.p.'s with

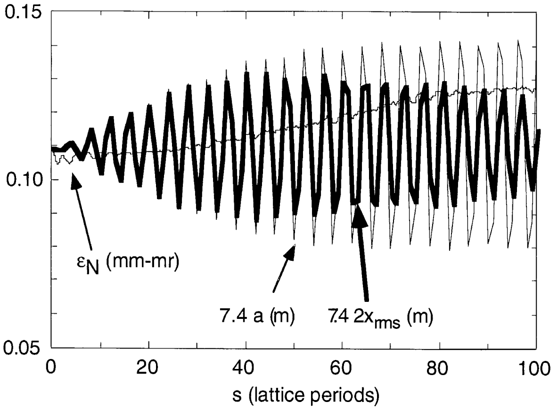

Fig. 1. Envelope parameter a, twice-rms beam size $2x_{\rm rms}$, both scaled by a factor of 7.4 for clarity, and normalized transverse emittance $\varepsilon_{\rm N} \equiv (\varepsilon_{{\rm N}x} + \varepsilon_{{\rm N}y})/2$, for beam driven near 1 : 1 resonance via 1% quadrupole error applied every four lattice periods.

$\sigma_0 = 74°$, $\sigma = 19.5°$ (parameters chosen to illustrate the effect especially clearly).

Coasting-beam emittance growth varies with proximity to resonance, though in all cases there is some growth because some mismatch is continually generated. For example, after an initial transient a 75 keV, 1.5 mA beam in a straight lattice undergoes about 7% emittance growth in 300 l.p.'s, as does an 80 keV, 1.75 mA beam. However, an 80 keV, 1.5 mA beam, being very close to the 5 : 1 resonance described above, suffers about 25% emittance growth over the same distance. All of these cases lead to the formation of a significant beam halo. When the perturbation to a 75 keV, 1.5 mA beam is applied every 10 l.p.'s (instead of every 20) the emittance growth over 600 l.p.'s is roughly halved, and the halo less extensive.

Power spectra of the single-particle x-coordinates were computed using the particle histories over an entire WARPxy simulation of an initially matched, coasting 75 keV, 1.5 mA beam with $\sigma_0 = 80°$ and $\sigma = 24°$, followed for 600 l.p.'s, and perturbed by a periodically applied 1% quadrupole error every 20 l.p.'s. The runs employed a 64×64 grid, 50 steps/l.p., and four-fold symmetry. These power spectra show considerable variation among particles; in general the spectra of particles with small oscillation amplitude peak at low frequencies (e.g. 15°/l.p.) because they see only a much-reduced net focusing field, while those of particles with large amplitudes peak at higher frequencies (e.g. 35°) because they sample the stronger net field at the beam edge.

The aggregated power spectra (computed by summing over the individual particle x-coordinate power spectra) are smooth curves which peak at 23°, rolling-off smoothly so as to be down by a factor of ~ 100 at 0° and 60°; there is finite power all the way out to 180°, where the spectrum was cut-off because orbits were sampled only once per l.p., in the middle of an "F" quad. Such aggregated power spectra computed over the final 200 l.p.'s are virtually the same as those over the initial 200 l.p.'s, except for a rough doubling of the power in the frequency range from 42° to 75°.

Power spectra of the beam's $x_{\rm rms}$ closely resemble those from the envelope calculations. An example is shown in Fig. 2, for a simulation following 80 k particles over 200 l.p.'s with the parameters above. There is a dominant peak at the alternating-gradient "flutter" frequency 360°, peaks at the mismatch mode frequencies 90° and 119° (the former being

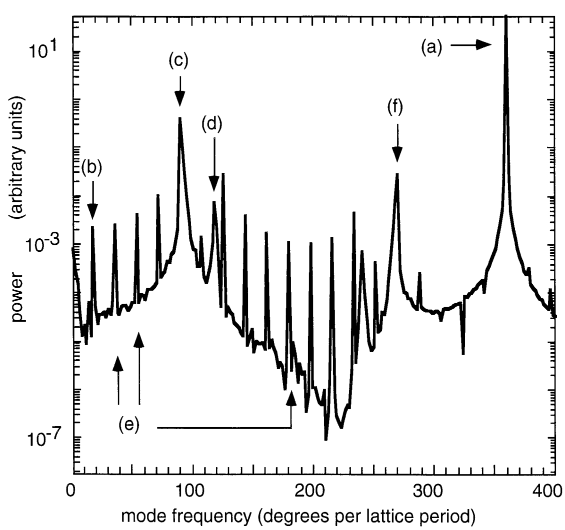

Fig. 2. Power spectrum of $x_{\rm rms}$ obtained from particle-in-cell simulation: (a) alternating-gradient (a–g) "flutter" mode at 360° per lattice period; (b) applied perturbation at 18°; (c) asymmetric mismatch mode at $\sim 90°$; (d) symmetric mismatch mode at $\sim 119°$; (e) harmonics of applied perturbation; (f) beat between a–g mode and asymmetric mismatch mode.

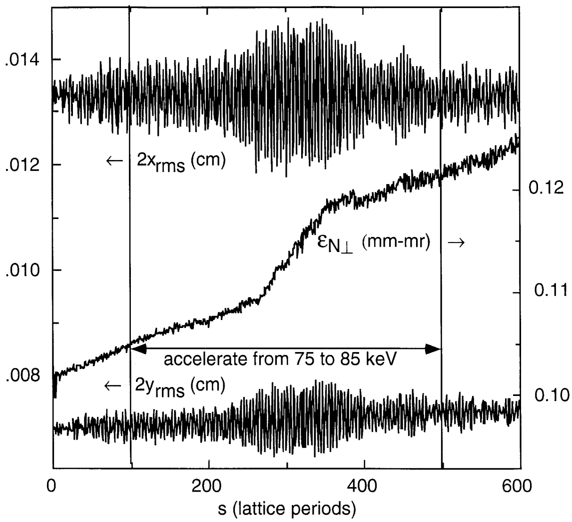

Fig. 3. Beam semi-axes $2x_{\mathrm{rms}}$ and $2y_{\mathrm{rms}}$ versus lattice period, and transverse normalized emittance (average of x and y emittances) for accelerating beam.

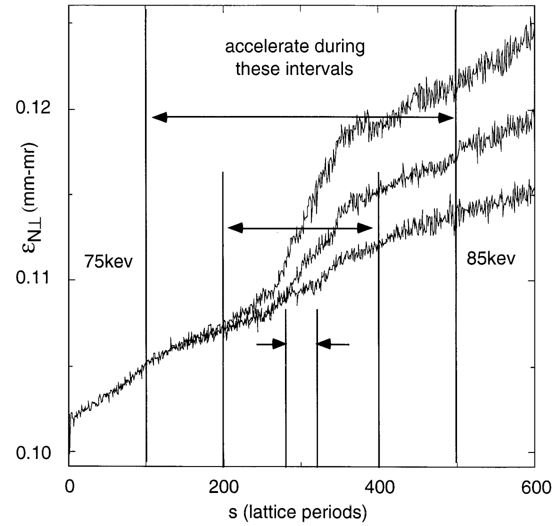

Fig. 4. Transverse normalized emittance (average of x and y emittances) for acceleration over various intervals. The lowest curve is associated with the shortest interval.

larger due to the resonant drive), a peak at the perturbation frequency 18°, and peaks at various harmonics and beat (difference) frequencies.

To examine passage through this resonance, we set a 75 keV beam coasting for 100 l.p.'s, then accelerated it over 400 l.p.'s to 85 keV, and finally allowed it to coast for 100 l.p.'s. Relatively slow, steady emittance growth was observed except between l.p. 260 and l.p. 350; see Fig. 3. This is consistent with resonantly driven growth, since the beam reaches 80 keV at l.p. 300. (This run used only 20 k simulation particles and so some of the emittance growth is collisional.) Note the correlation between the emittance growth rate and the instantaneous degree of mismatch. When the increase to 85 keV occurs more rapidly, the beam spends less time near resonance and the emittance growth is reduced. For acceleration over 200 l.p.'s the "excess" growth (over the non-resonant rate) is roughly halved, and for acceleration over 40 l.p.'s there is no discernible "excess" growth; see Fig. 4.

Similar suppression of emittance growth is observed when rapid bunching is carried out (simulated by increasing the charge of the simulation particles). When the current of a 75 keV beam is ramped from 1.3 to 1.7 mA over 200 l.p.'s there is a comparable enhanced emittance growth which is roughly halved when the bunching is done over 40 l.p.'s. However, a further decrease in the bunching time does not lead to any further reduction in the emittance growth. We attribute this to the fact that the change in current always leads to a similar downstream increase in beam size and attendant mismatch oscillation, since the space-charge-dominated beam with more charge is harder to confine.

3. Discussion

These simulations suggest that lattice errors can resonantly drive mismatch oscillations of the beam core, and then secondary resonances of particle motions with the beam core oscillations can parametrically drive particles into the halo. Acceleration or compression "through" resonances can have a mitigating effect.

This regime (very long runs, typically multiple small effects) is challenging for simulations, but careful selection of numerical parameters enables the examination of beams for many hundreds of l.p.'s at an affordable computational cost. Many of the runs used 20 k particles, a small enough number that there is significant numerically induced collisional emittance growth. To check, some cases were re-run using 80 and 320 k particles. Such runs behave much like those with 20 k particles but show somewhat less emittance growth; the 80 and 320 k runs are virtually identical to each other. The computed orbits occasionally show the effects of enhanced collisionality, but when 80 k or more particles are used they generally appear quite smooth; when "strobed" once per lattice period to suppress the rapid alternating-gradient motion, they precess smoothly around in the configuration space x, y.

These studies would benefit from a better emittance diagnostic, to help us distinguish halo formation from bulk emittance growth; for example, a diagnostic which computed the emittance from all of the particles would rapidly deviate from one using the "central" 80% of the particles when halo growth occurs. We are considering the implementation of such a measure.

This work has considered idealized problems and should be extended to include multiple, smaller errors; curvature effects; and quadrupole rotations, misalignments, and offsets all acting together all with realistically achievable tolerances. The 3-D model needs to be thoroughly exploited to explore coupling of motions between the different coordinate directions as a means of damping mismatch oscillations. Such couplings include a collective "equilibration effect" [7] as well as simple kinematic coupling associated with applied and self-fields.

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